DOI: 10.7251/EMC1902368B
Datum prijema rada: 25. oktobar 2019.
Datum prihvatanja rada: 12. decembar 2019.

UDK: 005.336:[656.7:510.5
Časopis za ekonomiju i tržišne komunikacije

Godina **IX** • broj **II**
str. 368-383

PREGLEDNI NAUČNI RAD / OVERVIEW SCIENTIFIC PAPER

EVALUATION OF UNCONSTRAINING METHODS IN AIRLINES' REVENUE MANAGEMENT SYSTEMS

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Abstract: *Airline revenue management systems are used to calculate booking limits on each fare class to maximize expected revenue for all future flight departures. Their performance depends critically on the forecasting module that uses historical data to project future quantities of demand. Those data are censored or constrained by the imposed booking limits and do not represent true demand since rejected requests are not recorded. Eight unconstraining methods that transform the censored data into more accurate estimates of actual historical demand ranging from naive methods such as discarding all censored observation, to complex, such as Expectation Maximization Algorithm and Projection Detruncation Algorithm, are analyzed and their accuracy is compared. Those methods are evaluated and tested on simulated data sets generated by ICE V2.0 software: first, the data sets that represent true demand were produced, then the aircraft capacity was reduced and EMSRb booking limits for every booking class were calculated. These limits constrained the original demand data at various points of the booking process and the corresponding censored data sets were obtained. The unconstrained methods were applied to the censored observations and the resulting unconstrained data were compared to the actual demand data and their performance was evaluated.*

Key words: *airline revenue management, demand forecasting, data censoring, unconstraining methods, Expectation Maximization Algorithm, Projection Detruncation Algorithm*

JEL: *C34, L93*

INTRODUCTION

Today's airline industry is facing many challenges while struggling to maintain marginal profits and continue the trend recorded in the last four years with a return on invested capital exceeding the industry's average cost of capital (Škurla

Babić, Ozmec-Ban, Bajić, 2018). The industry has made great efforts to develop sophisticated revenue management systems which would adequately respond to new requirements for forecasting demand and managing the availability of available seats in a competitive business environment.

Airline revenue management comprises a set of scientifically grounded strategic methods used by airlines to forecast fluctuating air travel demand and allocate seats in various fare classes with the aim of maximizing total flight revenue. A range of mathematical methods have been devised to calculate the booking limits of fare classes. With the development of information technologies, the computing power of processors and memory capacity no longer dictate the permitted degree of complexity and robustness of these methods. The focus of the problems of modern airline revenue management systems has, therefore, shifted to demand forecasting.

Based on the analysis of historical demand data, a demand-forecasting module generates the input data of the optimization module, that is, the forecasted demand parameters for each fare class. It is of vital significance to project the total (true) demand, which also includes rejected booking requests. Unconstraining methods use the recorded data of the realized demand to estimate the true demand for each fare class on different flights.

PREVIOUS RESEARCH

The first doctoral dissertation (Peter P. Belobaba, 1987) dealing with airline revenue management was published in 1987. Its research results are still used today in their original and author-modified form (EMSRa and EMSRb heuristic methods[1]). Several dissertations thereafter dealt with optimal leg-based and network seat inventory control policy in case of two, and later more, fare classes (Larry R. Weatherford, 1991; Dirk P. Günther, 1998; Darius Walczak, 2002). The issue of demand forecasting in the context of airline revenue management became the focus somewhat later and the first significant dissertation in the field which specified the issue of demand disaggregation and described passenger arrivals using the non-homogeneous Poisson model was published in 1990 (Anthony O. Lee, 1990). Later works shed more light on demand forecasting and emphasized its key role in the efficiency of airline revenue management systems and flight revenue maximization.

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1 EMSR (engl. Expected Marginal Seat Revenue) – EMSR models: EMSRa and EMSRb

William M. Swan (2002) and Richard Klophaus (2006) examined the methods of assessing true demand, the issue of describing demand using various probability distributions, and the problem of managing aircraft available seats under the terms of simplified tariff policy, respectively. It was noted that the normal probability distribution was not the best solution for describing demand probability in all cases, particularly in highest fare classes. Several AGIFORS[2] authors have dealt with the application of these methods in the context of airline revenue management (e.g., Stefan Pölt, 2000; Richard M. Ratliff et al., 2006; Catalina Stefanescu, 2009).

Weatherford and Pölt (2002) were the first to consider the Expectation–Maximization method in airline revenue management. They analyzed six unconstraining methods and concluded that the more the censoring exceeded 20 per cent, the poorer estimation results these naive methods would yield. At the same time, Expectation Maximization Method (EM Method) and Projection Detruncation Method (PD Method) proved to be the most efficient. In addition, they showed that the improvement of true demand forecast – switching from one of the naive methods to the EM or PD – leads to an increase in total flight revenue by 2–12 per cent. Zeni's PhD thesis evaluated several unconstraining methods which are used in practice. The possibility of using the EM algorithm was proposed in case a part of demand data does not exist (Richard H. Zeni, 2004).

In his later work, Weatherford tested the effect of using four unconstraining methods on total flight revenue. He concluded that a kind of optimization algorithm dictates various results when switching from one unconstraining method to another. His simulation results confirmed the importance of true demand forecasting and showed that upgrading the unconstraining process can lead to revenue gains of 2–15 per cent (Weatherford, 2012).

Alireza Nikseresht and Koorush Ziarati (2017) proposed unconstraining method that used Multinomial Logit model to model the customer choice behaviour. A simple algorithm was proposed to estimate the parameters (customers' preference) of the model by using historical sales data, product availability info and the market share. The proposed method was evaluated using different simulated datasets and the results showed that proposed method outperformed the others in terms of execution time and accuracy.

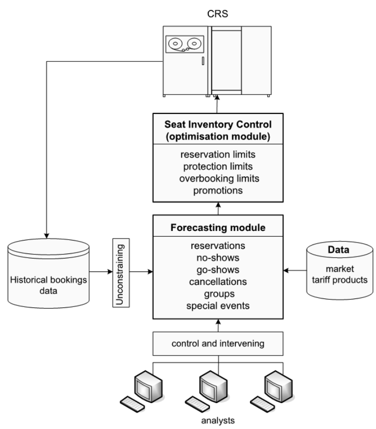
2 AGIFORS – Airline Group of the International Federation of Operational Research Societies, a professional association dedicated to applying operational research in solving air transport problems

Kourentzes et al. studied the frequently encountered situation of observing only a few sales events at the individual product level and proposed variants of small demand forecasting methods to be used for unconstraining (Nikolaos Kourentzes, Dong Li & Arne K. Strauss, 2019). Price et al. proposed an unconstraining method that used Gaussian process (GP) regression. They developed a novel GP model by constructing and implementing a new non-stationary covariance function for the GP which enabled it to learn and extrapolate the underlying demand trend. Their results indicate that GPs outperform existing single-class unconstraining methods (Ilan Price, Jaroslav Fowkes & Daniel Hopman, 2019).

THE ISSUE OF DEMAND CENSORING

The core of the airline revenue management system is seat inventory control module which distributes seats into fare classes by optimizing the expected flight revenue based on true demand parameters. They must be calculated using available historical data on realized demand, the demand that is frequently far away from actual demand. If a fare class demand or flight demand reaches the booking limit, the booking data will not represent the true demand but only a part of it. The number of requests rejected due to the closing of the fare class or flight is not recorded in airline reservation systems and cannot therefore be used as input for optimal seat allocation. Unconstraining methods are used to forecast total demand – the demand without capacity limitations. Figure 1 illustrates the process of a typical airline revenue management system.

Figure 1: Demand forecasting module in airline revenue management system



Source: Prepared by authors according to Kalyan T. Talluri & Garrett J. van Ryzin, 2005:19

The reservation system is used to continuously fill the database with historical bookings data. Unconstraining methods are used to truncate the data and obtain uncensored demand data. From those data, future demand parameters are estimated (mean μ and standard deviation σ) being simultaneously the output of forecasting module and the input of optimization module.

UNCONSTRAINING METHODS

Unconstraining methods are used to unconstrain, that is, increase the number of recorded requests by the number of requests which have been rejected due to capacity limitations and which have not been recorded in the airline's reservation system.

The aim of unconstraining truncated data is to use the recorded data (some of which are censored) to estimate the cumulative demand curve for each fare class. The demand is estimated at checkpoints[3] before the scheduled departure date, based on the information on the current number of confirmed bookings. For each checkpoint, the confirmed booking data are archived within the airline revenue management system, comprising thereby a historical database used for forecasting future flight demands. If all requests are accepted, that means no data were censored and the true demand corresponds to the recorded demand, that is, the number of bookings at a checkpoint. If some requests have been rejected, true demand data are truncated, and such censored data represent booking limits at certain checkpoints and not the true demand.

The following sections describe various unconstraining methods and assess their robustness and efficiency.

METHODS THAT USE ONLY AVAILABLE DATA

The methods that only use the available data do not attempt to replace the censored data with new values. There are two such methods. One ignores censoring (hereinafter I1 method) by simply disregarding the fact that part of the available recorded demand data does not represent true demand. The method uses all available data which also includes the ones archived in the system after a fare class has closed. The method that rejects censored values (I2) limits the set of available recorded demand data to the values which have not been censored.

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3 Syn. snapshot, review point, reading day.

Regardless of the simplicity and ease of implementation of these methods, their application in the demand-forecasting module in the airline revenue management system can only be justified if the quantity of censored data is extremely low.

METHODS THAT REPLACE CENSORED DATA WITH NEW VALUES

Methods that replace censored data with new values replace all the data on bookings in cases when fare classes were closed due to reached reservation limit with new values. The methods work in the following manner:
- if the seat availability indicator shows that at a checkpoint the fare class was "open", the recorded number of bookings represents true demand;
- if the seat availability indicator shows that at a checkpoint the fare class was "closed", the recorded number of bookings represents censored demand and is then replaced with the average uncensored value (RWA method), the median (RWM method), the upper quartile (RWP75) or any other percentile, as long as these values are higher than the ones recorded.

STATISTICAL UNCONSTRAINING METHODS

The statistical methods used for true demand forecasting require more complex calculations which often comprise several iterations and demand more time for estimation and a powerful computer support.

The Booking Profile method (hereinafter BP method) determines true demand based on the booking profile curve and starts from the assumption that for similar flights[4], the booking profile curve of a fare class does not depend on demand intensity – the increase between adjacent checkpoints is constant for a set of similar flights. This means that for similar flights, the booking profile curve for fare classes can be accurately estimated by averaging the demand at every checkpoint. Uncensored booking data, which probably represent flights with lower demand, are averaged at every checkpoint and increase coefficients are calculated for every two adjacent checkpoints. The censored data are then unconstrained so that the last uncensored item of data at a previous checkpoint is increased by multiplying the calculated increase coefficients.

The EM algorithm (Expectation Maximization algorithm) is a general-purpose algorithm used for estimating maximum likelihood of parameters of truncated

[4] In the context of airline revenue management, a set of similar flights includes flights for which the number of bookings does not considerably exceed the average booking number for other flights in the same set.

data distribution. It is used for calculating the set of parameters that describe the hidden probability distribution when only a part of data is available. Its use is justified by the fact that in many statistical applications that deal with censored data the estimation of the maximum likelihood parameters is made difficult by the structure of the corresponding (log) likelihood function and the direct optimization over the incomplete-data (log) likelihood function tends to be a difficult-to-solve maximization problem (Tudor Bodea & Mark Ferguson, 2014).

It is assumed that there are $M + N$ booking data for a fare class, where M are censored data and N values represent true demand. In the iterative algorithm, the EM method uses the highest probability function L (μ, σ, $M + N$) so the E-step (estimation) and M-step (maximization) alternate with each other. All the censored values in the E-step are replaced with their estimated values (conditional estimation) by using the current hypothesis parameters on normal distribution, μ and σ. Then, in the M-step, the highest probability function lnL (μ, σ, $M + N$) is maximized based on the corrected values and new values are estimated for μ and σ. These steps alternate with each other until μ and σ obtained through iterations start to converge.

The Projection Detrunction Method (PD) is essentially similar to the EM algorithm. It is used in the PODS simulator and was developed at Boeing thanks to C. Hopperstad. The algorithm starts from the assumption of normal demand distribution and first calculates the average value of the number of requests for "open" flights. The algorithm then uses the arbitrary value τ for estimating demand on "closed flights". The true demand on a closed flight is replaced by the new value so that the ratio between the area below the normal distribution curve to the right of the new value and the area right to the original value equals τ (Craig H. Hopperstad, 1997). The process is repeated for the "closed" flights until the forecast values (both the mean and standard deviation) begin to converge. The τ constant impacts the unconstraining aggression, and for lower τ values, the PD algorithm will generate higher values of estimated demand.

UNCONSTRAINING METHODS COMPARISON

The comparison of various unconstraining methods performance assumes the availability of true, uncensored demand. Without the possibility of comparing the parameters of forecasted demand and the parameters of true demand it is impossible to assess the efficiency of an unconstraining method.

In his doctoral dissertation, Richard H. Zeni carried out a simulation of a situation without any rejected requests because the aircraft capacity was sufficient enough. He then analyzed what would have happened had the capacity been smaller that it was (Zeni, 2001). Data censoring simulations used in this paper are the closest to the ones described by Zeni's approach. Apart from generating demand and input data for a program which calculates in estimation errors and compares the accuracy of certain methods, the underlying difference in this paper is the fact that the optimization model used for calculating booking limits is EMSRb, which is the industry standard, whereas Zeni had used EMSRa.

DEMAND DATA CENSORING SIMULATION

To generate comparable sets of true and censored demand, which will be used for the comparison of unconstraining methods, a situation was simulated in which the aircraft capacity is much higher than true demand and in which there can be no rejected booking requests in any of the fare classes. Afterwards, the aircraft capacity was reduced, which led to requests being rejected due to booking limits. True demand data were compared to the recorded demand data in case of the reduced capacity in order to establish cases in which data are censored.

ICE V.20, a software developed by *The Boeing Company*, was used in this paper to generate demand data. Four basic true demand scenarios were simulated, each comprising 771 iterations (flights), with set simulation parameters which also included aircraft capacity, defined so as not to reject requests at any given moment. Since a special optimization is carried out for every part of the cabin, five economy classes were defined with tariffs ranging from 770 to 340 and with the average demand of 10, 20, 30, 40, 50 and 80 for classes 1 to 5.

Eleven checkpoints were used in all simulation sets. Demand set simulations used two combinations of cumulative booking curves. The first combination (hereinafter, T reservation curves) assumes characteristic (theoretical) booking curves which are concave for lower fare classes and convex for higher ones. The second combination (hereinafter CA booking curve) was made based on the analysis of bookings on 36 similar Croatia Airlines domestic flights. The original data was distributed in fourteen price ranges for 24 sub-intervals and grouped to obtain booking curves with 10 sub-intervals and 5 fare classes. The result of the data processing was relatively unexpected: the booking curves were not particularly concave for higher fare classes nor were they convex for lower classes. The

CA booking curves were only slightly concave or convex and all the curves were close to the provisionally homogeneous curve.

The average K-factor or variation coefficient is the ratio of the standard deviation to the mean, reflecting the imminent demand variability. The simulation sets were modeled so as to differentiate between high and low K-factors. In the simulation, in the sets marked with H (high K-factor) the total K-factor was 0.35, ranging from 0.5 for the first class to 0.4 for the lowest fare class. In the simulation sets marked L (low K-factor), the total K-factor was 0.2, ranging from 0.4 for the first class to 0.25 for the lowest fare class.

For each of the four basic demand sets obtained (CA_H, CA_L, T_H, and T_L), four simulation sets of aircraft cabin loading were performed (with 771 iterations) so the capacities were limited to 132, 164, 196, and 228 seats. This produced 16 sets of cabin loading, in which requests were rejected and demand censored. However, each of these sets had its own corresponding loading set without censoring. Therefore, 16 pairs of simulation sets were modeled, with each pair including a set which had rejected requests and censoring and a set in which all the requests had been accepted and whose data represented true demand. The number of accepted and rejected requests for 16 simulation sets with limited aircraft capacity is shown in Table 1.

Table 1: Accepted and rejected requests in simulated data sets

Simulated data set	requests (average)	Class 1	Class 2	Class 3	Class 4	Class 5	Total	Average flight revenue
CA_H_132	accepted	9,26	19,09	27,34	43,40	26,41	125,50	61 041,40
	rejected	0,91	1,22	3,26	7,53	54,74	67,65	
CA_H_164	accepted	9,51	19,40	28,17	46,68	45,73	149,48	69 826,47
	rejected	0,66	0,91	2,43	4,25	35,43	43,67	
CA_H_196	accepted	9,70	19,67	28,91	48,38	61,37	168,03	76 576,41
	rejected	0,46	0,64	1,69	2,55	19,79	25,12	
CA_H_228	accepted	9,90	19,92	29,54	49,63	71,56	180,55	81 210,17
	rejected	0,26	0,39	1,06	1,30	9,59	12,60	
CA_L_132	accepted	8,92	19,04	27,39	44,31	30,93	130,60	62 705,07
	rejected	0,99	1,21	2,88	5,54	48,78	59,40	
CA_L_164	accepted	9,17	19,36	28,25	46,62	55,03	158,43	72718,88
	rejected	0,75	0,89	2,02	3,23	24,69	31,58	

CA_L_196	accepted	9,52	19,81	29,21	48,47	70,61	177,62	79 855,94
	rejected	0,40	0,45	1,06	1,37	9,11	12,39	
CA_L_196	accepted	9,77	20,09	29,90	49,48	77,45	186,69	83 342,44
	rejected	0,15	0,17	0,37	0,36	2,27	3,31	
T_H_132	accepted	8,84	17,91	27,82	43,02	26,88	124,46	60 172,55
	rejected	1,17	1,73	2,68	7,15	54,82	67,54	
T_H_164	accepted	9,10	18,24	28,44	46,22	45,86	147,87	68 736,71
	rejected	0,91	1,40	2,06	3,94	35,83	44,14	
T_H_196	accepted	9,37	18,66	29,12	47,78	63,10	168,02	75 361,73
	rejected	0,64	0,98	1,38	2,38	18,60	23,98	
T_H_228	accepted	9,59	20,00	29,59	48,72	72,05	178,94	80 162,11
	rejected	0,42	0,64	0,91	1,45	9,65	13,07	
T_L_132	accepted	8,55	17,97	27,41	44,57	31,69	130,19	62 086,28
	rejected	1,32	1,80	2,46	5,21	47,98	58,77	
T_L_164	accepted	8,88	18,36	28,07	46,42	56,01	157,74	71 991,56
	rejected	0,99	1,41	1,80	3,36	23,66	31,22	
T_L_196	accepted	9,31	18,98	28,96	48,31	71,20	176,76	79 158,42
	rejected	0,55	0,80	0,91	1,47	8,47	12,19	
T_L_228	accepted	9,66	19,52	29,58	49,38	77,41	185,55	82 662,65
	rejected	0,21	0,26	0,29	0,39	2,26	3,41	

Source: Authors

The parameters of set simulations of the same pair differ only in aircraft capacity. All other indicators of demand, tariffs, cabin configuration and seat allocation policy, which must be defined in order to generate the simulation set, were identical. The censored values in 16 data sets were truncated by using eight unconstraining methods described earlier. Their precision was established by comparing unconstrained and actual demand parameters, which were calculated using the unconstrained values and true demand values.

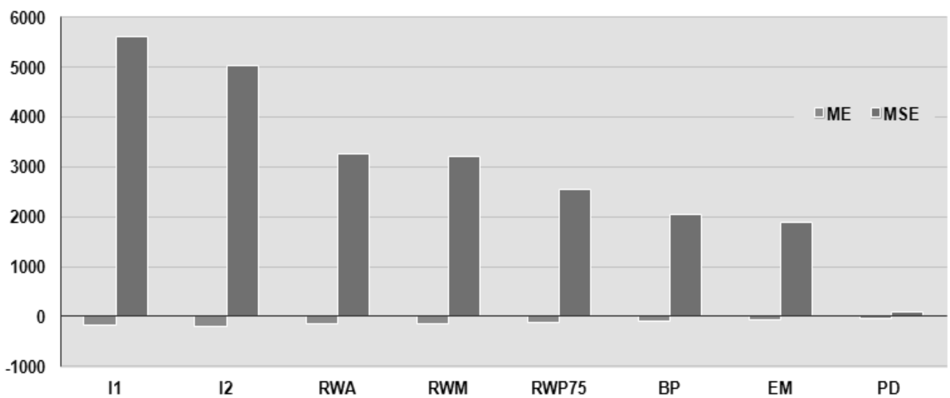
Airline revenue management systems forecast future flight demand based on a number of similar flights. This paper defined future demand by looking at 26 historical flights. The number was chosen because it reflects a half-year period under the assumption that a day of the week is a relevant determinant in defining similar flights. In each of the 16 scenarios, a set of 771 iterations includes 150 groups of 26 iterations which comprise iterations from $1 + k \cdot 5$ to $26 + k \cdot 5$, $k = 0.....149$. Therefore, the groups include iterations 1–26, 6–31, 11–36, …, 746–771. For the 150 groups, unconstrained demand parameters were estimated (mean and standard deviation) based on the unconstrained demand data using unconstraining methods. Analogous thereto, for each of the 16 scenarios, true demand parameters were calculated from the 771 samples of true demand for the 150 groups.

After that, it was possible to compare unconstrained demand parameters (calculated using various unconstraining methods) and true demand parameters. To carry out the comparison of unconstraining methods an application was developed using Visual Basic. The input data of the application were values of true demand parameters, i.e. output values of the ICE simulation software, as well as unconstrained demand parameters for 16 pairs of simulation sets. The Mean Error (ME) and Mean Square Error (MSE) were calculated for each method and each scenario of demand.

THE ANALYSIS AND COMPARISON OF ACCURACY OF UNCONSTRAINING METHODS

Based on the results of the simulations of aircraft loading and the unconstraining of censored demand data for the defined sets of data, this section compares the accuracy of eight unconstraining methods described in chapter 3. Illustrated in Figure 2 are mean errors (ME) and mean square errors (MSE) of mean estimates for 8 unconstrained methods that were tested. Table 2 lists the MSE values of mean estimates for all unconstraining methods at all checkpoints.

Figure 1: Total ME and MSE of mean estimates for all unconstraining methods



Source: Authors

The I1 method ignores censoring and does not take any actions so as to unconstrain the demand data. The application of the I2 method enlarges the error compared to the I1 method because the estimated mean value is – in case of rejecting censored values – lower that the estimated mean value based on the set which also comprises the censored data (I1 method).

The methods that unconstrain the censored values by replacing the maximum between the values and the average, the median, or the 75th percentile of uncensored set values yield similar results. Their use is simple and requires only a fraction of the time that statistical methods require.

Statistical methods ensure the best results meaning that they produce the smallest ME and MSE errors which were used to measure the accuracy of unconstraining methods. Such a result is consistent with the previous scientific research. In fact, these methods are regularly used in complex airline revenue management systems by leading world airlines.

Table 2: MSE of mean estimates for each unconstraining method at all review points

method	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	total
I1	7,5	55,4	127,2	234,1	317,4	454,5	594,7	746,2	1 017,5	1 026,9	**5 612,5**
I2	5,4	31,5	76,9	149,9	213,0	346,9	474,8	657,9	964,2	1 044,5	**5 035,9**
RWA	3,4	23,9	58,9	113,5	156,4	243,6	328,1	435,0	622,7	630,7	**3 249,1**
RWM	3,6	24,3	58,6	112,5	153,6	241,8	323,5	430,1	615,6	623,5	**3 213,2**
RWP75	2,0	15,0	40,2	79,7	110,9	181,2	250,6	343,3	503,3	510,0	**2 548,0**
BP	3,4	44,3	84,1	132,9	152,9	202,4	240,8	265,9	308,6	309,6	**2 055,1**
EM	1,6	13,6	34,7	66,3	90,9	137,4	188,8	247,2	358,5	372,0	**1 892,2**
PD	1,2	9,7	24,9	47,5	65,4	99,0	138,1	186,6	282,7	310,6	**1 492,5**

Source: Authors

Using the BP method to unconstrain data ensures that the estimated value is higher than the maximum value in the group of historical bookings, which is also its advantage. On the other hand, estimated values are sometimes very high and can lead to true demand overestimation. The EM algorithm and PD method yield the best results, as expected. The advantage of the PD method is its ability to adjust unconstraining levels by changing the τ constant.

Table 3 lists the ME and MSE values of mean estimates for all unconstraining methods with regard to the booking curves and K-factor. On the whole, the larger the demand K-factor, the more inaccurate the mean estimation, that is, the higher the forecast errors. Furthermore, the data sets with theoretical booking curves recorded higher values of forecasting errors compared to the CA booking curves, which can be simply explained by the larger number of censored values for the same demand level and the earlier closing of lower fare classes.

Table 3: ME and MSE of mean estimates for various K-factors and booking curves

data set	CA_H		CA_L		T_H		T_L	
method	ME	MSE	ME	MSE	ME	MSE	ME	MSE
I1	-181,33	5 705,70	-116,60	3 329,08	-259,70	8 555,59	-168,38	4 859,69
I2	-199,03	4 738,46	-118,33	2 871,79	-307,87	7 735,74	-182,01	4 797,50
RWA	-139,50	3 284,57	-84,45	1 808,38	-207,12	5 188,26	-124,92	2 715,21
RWM	-138,33	3 228,39	-84,16	1 802,81	-205,52	5 104,04	-124,67	2 717,67
RWP75	-112,79	2 388,85	-70,15	1 546,80	-171,19	3 878,96	-105,62	2 377,45
BP	-89,19	461,44	-70,19	993,00	-153,73	2 482,85	-60,38	4 283,15
EM	-70,04	1 930,48	-45,47	1 110,25	-96,86	2 976,19	-63,17	1 551,91
PD	-50,98	1 502,78	-33,63	910,24	-69,79	2 321,87	-46,06	1 235,04

Source: Authors

Table 4 shows forecasting errors of mean estimates for various aircraft capacities, that is, for different censoring levels.

Table 4: ME and MSE of mean estimates for various aircraft capacities

data set	C=132		C=164		C=196		C=228	
method	ME	MSE	ME	MSE	ME	MSE	ME	MSE
I1	-392,51	15 761,92	-205,78	5 113,98	-91,73	1 293,34	-35,99	280,82
I2	-360,63	12 324,24	-230,41	4 863,76	-140,64	2 152,63	-75,56	802,85
RWA	-278,86	8 489,72	-162,12	3 232,85	-81,25	1 023,89	-33,76	249,96
RWM	-277,73	8 415,65	-160,79	3 180,69	-80,57	1 009,90	-33,58	246,68
RWP75	-238,38	7 016,61	-128,64	2 281,63	-64,60	711,13	-28,13	182,69
BP	-237,73	7 526,22	-78,63	518,02	-43,79	138,12	-13,35	38,09
EM	-172,80	5 520,47	-74,45	1 660,68	-22,88	327,78	-5,41	59,90
PD	-129,08	4 367,37	-52,01	1 283,92	-15,63	264,96	-3,73	53,68

Source: Authors

Forecasting error values are higher for lower aircraft capacity, i.e. for higher percentage of censored values in data sets used for true demand forecasting.

Although all previous tables have listed and taken into consideration errors in the lowest fare class, it is important to notice that the airline revenue management system that uses the EMSRb model in the optimization module does not at all require any demand estimation data for the lowest fare class. Therefore, the forecasting error values for the lowest fare class can be omitted when comparing the results of various methods because these estimations are ultimately not used. As in lower fare classes, due to the nested booking limits in revenue management systems, the most intensive censoring occurs, that can significantly affect the overall result and the accuracy of certain methods. So, it is justified to completely disregard forecasting errors in the lowest fare class. Table 4 lists the forecasting errors for the two most robust unconstraining methods, which have shown to be most accurate in estimating true demand values.

Table 4: ME and MSE of mean estimates for the EM and PD methods

klasa / metoda		1	2	3	4	5	U
EM	ME	0,81	2,04	4,10	10,58	-86,42	-68,88
	MSE	0,74	4,20	12,75	53,22	1 821,29	1 892,21
PD	ME	1,02	2,44	5,04	14,20	-72,82	-50,11
	MSE	1,18	6,23	20,96	103,47	1 360,65	1 492,48

Source: Authors

The PD method (τ = 0.4) yields the lowest forecasting errors if the data on recorded demand for all fare classes are taken into consideration. However, if the lowest fare class data are eliminated, that ceases to be the case, so the EM method is more accurate and ensures a higher quality of censored data unconstraining. The reason for that is the use of the unique aggression constant τ for all data groups within a simulation set, which corresponds to the practice among airlines.

CONCLUSION

As the true demand data are an essential input parameter for the optimization module of the airline revenue management system, censored demand data must be unconstrained to forecast total demand – the demand if there were no booking limits. For that purpose, several methods have been devised to unconstrain demand, in which the truncated demand data are at all checkpoints increased by the number of requests that have, due to the booking limits, initially been rejected. This paper has specified eight known unconstraining methods, from the simplest ones which use only available data to the complex statistical methods which use an algorithm to estimate maximum probability distribution of truncated data.

Unconstraining methods were analyzed using the data obtained by simulating 16 demand scenarios and 32 aircraft loading scenarios (each with 771 iterations). In the first 16 simulation sets the aircraft capacity was significantly bigger than the true demand which did not lead to requests being rejected in any fare class. For each of such flights generated in the first 16 data sets, the corresponding flights in the next 16 simulation sets were generated. They had the same demand scenario, but aircraft capacity was reduced and that led to closing of some fare classes and rejecting passengers, thus censoring demand. The censored values were unconstrained by using eight unconstraining methods and the obtained results were compared to the corresponding data on true demand.

Simulating the process of true demand data censoring has yielded credible data sets for testing unconstraining methods. For quantifying the results of some methods, a program was created using Visual Basic. For the eight unconstraining methods, the values of estimated demand parameters and values of true demand parameters were compared and relevant forecasting errors were calculated. The expectation maximisation and projection detruncation methods produced the most accurate forecasts and the advantage of the former method is its ability to adjust the degree of unconstraining by using the τ constant.

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